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***Corresponding Author:**

Jung-Woo Kim

Dept. of Naval Architecture, Republic of Korea Naval Academy
1 Jungwon-ro, Jinhae-gu, Changwon-si, Gyung-sangnam-do, 51704, Republic of Korea
Tel: +82-55-969-5313
E-mail: jwkim10616@navy.ac.kr

Performance Enhancement of Piezoelectric Energy Harvesters via Geometrical Structural Tuning

기하학적 구조 조율을 통한 압전 하베스터 성능 향상

Seong-Jin Lee¹, Jung-Woo Kim^{2*}

¹Researcher, Aerodynamics and Performance Research Team, Korea Aerospace Research Institute

²LTJG, ROK Navy/Assistant professor, Dept. of Naval Architecture, Republic of Korea Naval Academy

이성진¹, 김정우^{2*}

¹한국항공우주연구원 공력성능팀 연구원

²해군 중위/해군사관학교 조선공학과 조교수

1. Introduction

Piezoelectric materials, as anisotropic dielectrics, exhibit the unique ability to convert mechanical deformation into electrical signals and vice versa. Among them, ceramic-based materials such as lead zirconate titanate (PZT) have been widely employed in vibration based energy harvesting systems due to their high piezoelectric coefficients, thermal stability, and fabrication feasibility. However, their electrical output characteristics are highly sensitive to a multitude of factors, including crystal symmetry, doping composition, boundary constraints, and mechanical loading conditions [1]. For practical application, these parameters must be quantitatively characterized and reproducible under defined conditions [2].

In response to these demands, the US Navy established a

Abstract

This study investigates how auxetic substrate geometry affects the performance of cantilever-type piezoelectric energy harvesters. Based on Hamilton's principle, we derive an electromechanical model linking curvature-induced strain to electrical output. The model reveals that a negative Poisson's ratio enhances trace strain via in-plane deformation alignment, resulting in stronger electric fields. Finite element simulations using identical external geometries confirm reduced resonant frequency and amplified local stress. The auxetic harvester shows doubled voltage and tripled power, solely from geometrically induced strain amplification. These findings demonstrate that structural design—when guided by field theory—can act as a multiplier for energy harvesting, enabling lighter and more adaptive systems without material changes. In this light, the beauty of materials science lies not merely in composition but in geometrical tuning.

본 연구는 어그제틱 기판 구조가 캔틸레버형 압전 하베스터의 성능에 미치는 영향을 분석하였다. 해밀턴 원리에 기반한 모델은 곡률 유도 변형률과 전기 출력 간의 연관성을 정량화하며, 음의 푸아송비가 면내 변형 정렬을 통해 전기장을 증폭시킨다는 것을 보인다. 동일 외형의 구조를 비교한 유한요소해석 결과, 공진 주파수는 낮아지고 응력은 집중되었으며, 전압과 전력은 각각 2배 및 3배 향상되었다. 본 연구는 구조 설계가 물리장 기반 이론에 따라 에너지 하베스팅의 증폭기로 작용할 수 있으며, 재료나 질량의 변화 없이도 더 가볍고 적응적인 시스템 구현이 가능함을 보여준다. 이러한 관점에서 재료과학의 미학은 조성에 있는 것이 아니라 형상의 정밀한 조율에 있음을 나타낸다.

Keywords

압전 에너지 수확(Piezoelectric Energy Harvesting), 구조 기반 증폭(Structure-induced Amplification), 곡률 조율(Curvature-driven Tuning), 해밀턴 원리 기반 해석(Hamiltonian-based Analysis), 유한요소해석(Finite Element Analysis)

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standardized classification system in the mid-20th century to incorporate PZT materials into military grade devices such as underwater acoustic sensors, impact detectors, and communication equipment [3]. This system, known as the Navy Type I–VI classification, was grounded in key material parameters including elastic modulus, dielectric constant, piezoelectric coefficients (d_{31} , d_{33}), and mechanical quality factor (Q_m), to enable systematic material selection aligned with specific operational requirements. By doing so, the Navy ensured the reliability and performance consistency of piezoelectric components in mission-critical environments, transforming material selection from empirical practice into a structured, data-driven engineering discipline.

In parallel, the evolution of modern naval platforms, including unmanned surface vehicles (USV) and long-duration autonomous sensing systems, has introduced new challenges for sustained power supply under harsh oceanic conditions. In such environments, where battery replacement or external power input is impractical, self-powered mechanisms have become increasingly vital [4]. Among the various strategies available, piezoelectric energy harvesting emerges as a particularly promising solution, owing to its ability to convert ambient vibrations, hydrodynamic forces, or structural deformations directly into usable electrical energy [5].

Cantilever type piezoelectric harvesters remain a popular architecture due to their simplicity in design and the ability to be tuned to specific resonant frequencies [6]. Yet, the inherent limitations in output power under fixed material constraints call for structural optimization approaches that can enhance energy conversion efficiency without altering the base material itself. One compelling strategy lies in the implementation of auxetic structures—lattices char-

acterized by a negative Poisson's ratio—which respond to mechanical strain in a fundamentally different way [7]. By expanding rather than contracting under tension, auxetic geometries create favorable conditions for localized stress concentration and increased curvature, both of which contribute to amplified voltage output via curvature-dependent terms in the piezoelectric constitutive equations [8,9].

In this study, we propose and evaluate a hybrid energy harvester that combines PZT-5H piezoelectric layers with an integrated auxetic lattice structure. Using finite element analysis (FEA), we simulate and compare the voltage output of a bare cantilever and an auxetic embedded cantilever at their respective first-mode resonant frequencies. The results provide a quantitative basis for understanding the relationship between geometry and electrical performance, demonstrating a measurable improvement in output due to structural amplification effects.

Moreover, the analysis framework extends beyond empirical comparison by introducing an effective property model that formalizes the interplay between geometric design and electromechanical response. This approach allows us to capture the emergent behavior of the system as a function of both material properties and structural configuration. In doing so, we highlight a design pathway for maximizing energy harvesting efficiency in naval applications, one that relies not on changing what the material is, but on remaining what the material does.

2. Theoretical Derivation

We begin from the foundational principle governing the dynamics of elastic continua, namely Hamilton's principle, which states that the integral over time of the Lagrangian variation must vanish for all admissible variations of the displacement field. That is,

$$\delta \int_{t_1}^{t_2} (T - U) dt = 0 \quad (1)$$

where T is the total kinetic energy and U the total potential energy stored in deformation. For a one-dimensional cantilever beam with transverse displacement field $w(x,t)$, having effective flexural rigidity $E(x)I(x)$, area $A(x)$, and density $\rho(x)$, we express the kinetic and potential energies respectively as

$$T = \frac{1}{2} \int_0^L \rho(x) A(x) \left(\frac{\partial w}{\partial t} \right)^2 dt \quad (2)$$

$$U = \frac{1}{2} \int_0^L E(x) I(x) \left(\frac{\partial^2 w}{\partial x^2} \right)^2 dx \quad (3)$$

The resulting Euler-Lagrange equation yields the governing PDE of motion:

$$\rho(x) A(x) \frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial x^2} \left[E(x) I(x) \frac{\partial^2 w}{\partial x^2} \right] = 0 \quad (4)$$

Assuming harmonic base excitation and modal expansion of the form $w(x,t) = \phi(x) \cos(\omega t)$, the governing equation becomes a self-adjoint eigenvalue problem:

$$\frac{d^2}{dx^2} \left[E(x) I(x) \frac{d^2 \phi}{dx^2} \right] = \rho(x) A(x) \omega^2 \phi(x) \quad (5)$$

The first natural frequency $f_1 = \omega_1/2\pi$ arises as the smallest eigenvalue of this operator, and is critical for maximizing the efficiency of energy harvesting, as the system operates at resonance.

In the context of a multilayer cantilever composed of a substrate, intermediate elastic layer, and a surface mounted piezoelectric layer, the flexural rigidity and mass per unit length are no longer constants, but can be homogenized by using effective properties. Specifically, we denote the effective bending stiffness as

$$EI_{eff} = \sum_i E_i I_i + \sum_i E_i A_i (z_i - z_n)^2 \quad (6)$$

where z_n is the neutral axis of the composite section, and $I_i = b_i t_i^3/12$ is the second moment of area of layer i . Likewise, the effective density is defined as

$$\rho_{eff} = \sum_i \rho_i \frac{A_i}{A_{tot}} \quad (7)$$

The introduction of auxetic architecture into the substrate layer alters both E_{eff} and ρ_{eff} through the presence of pores or internal voids, quantified via the porosity parameter $\phi \in [0,1]$, such that

$$E_{eff} = E_0(1 - \phi)^n \quad (8)$$

$$\rho_{eff} = \rho_0(1 - \phi)^n \quad (9)$$

The exponent $n \in [1,3]$ accounts for structural sensitivity of stiffness to porosity, which may vary depending on unit cell geometry.

For the piezoelectric layer located at distance z_p from the neutral axis, the bending strain under Euler–Bernoulli assumption is

$$\varepsilon_{xx}(x) = -z_p \frac{d^2 \phi}{dx^2} \quad (10)$$

Due to transverse isotropy and the influence of Poisson coupling, the lateral strain is

$$\varepsilon_{yy}(x) = -\nu_{eff} \varepsilon_{xx} \quad (11)$$

and the resulting in-plane trace strain becomes

$$\varepsilon_{trace}(x) = \varepsilon_{xx}(x) + \varepsilon_{yy}(x) = (1 - \nu_{eff}) \varepsilon_{xx}(x) \quad (12)$$

Here, the introduction of auxetic structure with $\nu_{eff} < 0$ yields an increase in trace strain, a key driver of electromechanical enhancement.

From the constitutive theory of piezoelectricity, the linear coupling of mechanical and electrical fields is described by

$$S_{ij} = s_{ijkl}^E T_{kl} + d_{kij} E_k \quad (13)$$

$$D_i = d_{ijk} T_{jk} + \varepsilon_{ik}^T E_k \quad (14)$$

where S_{ij} is strain T_{kl} stress, D_i electric displacement, E_k electric field, d_{ijk} piezoelectric coupling tensor, s_{ijkl}^E compliance at constant electric field, and ϵ_{ik}^T permittivity at constant stress.

For the present case of a surface-mounted transversely isotropic piezoelectric layer under open-circuit condition and dominant 1D axial stress, the simplified relation becomes

$$D_3 = d_{31}\sigma_{xx} + \epsilon_{33}E_3 = 0 \tag{15}$$

Eq. (15) is equal to $E_3 = -d_{31}\sigma_{xx}/\epsilon_{33}$. With $\sigma_x = E_p\epsilon_{xx}$, and recalling the strain relation above, the electric field across the thickness of the piezoelectric layer is

$$E_3(x) = \frac{d_{31}E_p z_p}{\epsilon_{33}} \frac{d^2\phi}{dx^2} \tag{16}$$

and the induced open-circuit voltage is

$$V = \int_0^{t_p} E_3(x) dz = \frac{d_{31}E_p z_p t_p}{\epsilon_{33}} \frac{d^2\phi}{dx^2} \tag{17}$$

This reveals that the second spatial derivative of the transverse mode shape—i.e., the local curvature—at the location of the piezoelectric layer directly determines the output voltage.

Finally, the average power harvested over one oscillation cycle through an impedance-matched resistive load, modeled via the capacitive approach, is given by

$$P = \frac{1}{2} C_p V^2 f \tag{18}$$

$$C_p = \epsilon_{33} \left(\frac{bl_p}{t_p} \right) \tag{19}$$

leading to the expression:

$$P(x) = \frac{1}{2} \left(\frac{d_{31}E_p z_p t_p}{\epsilon_{33}} \right)^2 \left(\frac{bl_p}{t_p} \right) \left(\frac{d^2\phi}{dx^2} \right)^2 f \tag{20}$$

Thus, the output power is quadratically dependent on the curvature at the piezoelectric site, and is modulated multiplicatively by the flexural stiffness and strain-aligned effects of the substrate geometry.

This mathematical derivation explains our observed result that auxetic geometries with increasing unit cell counts lead to lower resonant frequencies (due to reduced E_{eff}), yet simultaneously yield significantly greater output power at resonance (due to enhanced curvature and strain alignment). In particular, the empirical relation between the number of unit cells and output power—although heuristic in appearance—is in fact a direct consequence of the structural-softening and energy-focusing effects embedded in the derivation above.

3. Results

To highlight the mechanical advantage of auxetic substrates, Fig. 1 illustrates the fundamental difference in strain behavior between conventional and auxetic structures under tensile loading. Conventional materials, characterized by a positive Poisson’s ratio, contract laterally when stretched axially. In contrast, auxetic structures exhibit the opposite trend, expanding laterally due to their re-entrant geometry, thereby demonstrating a negative Poisson’s ratio.

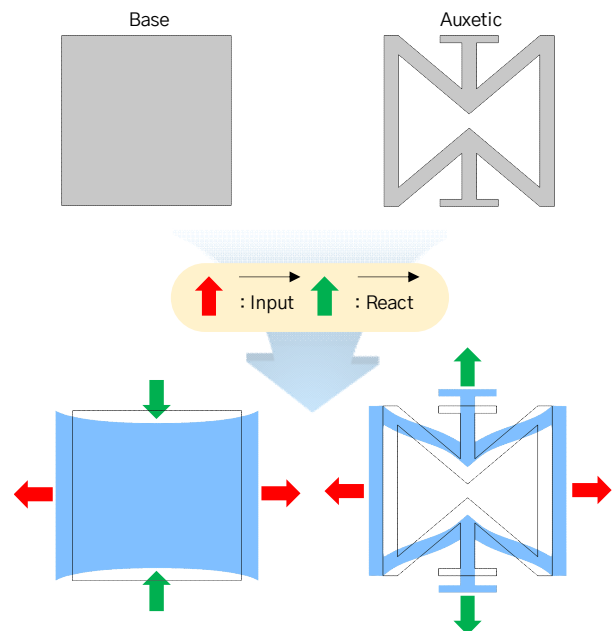


Fig. 1. Comparison between conventional and auxetic structures under tensile loading

This unique mechanical response has significant implications for piezoelectric energy harvesting, as it enables ε_{11} and ε_{22} to share the same sign, enhancing the trace strain ($\varepsilon_{11} + \varepsilon_{22}$) and thus increasing the open-circuit voltage output, as derived in the previous section.

To move beyond idealized formulations, we implemented a finite element model that captures the electromechanical behavior of the system under realistic conditions. The goal was not merely to validate the theoretical predictions, but to interrogate how geometry reshapes the field of strain in a tangible, simulated construct.

Fig. 2 outlines the numerical setup. The model consists of a multilayer cantilever with a fixed boundary at one end, subjected to base excitation. A piezoelectric patch (PZT-5H) is surface-mounted near the clamped edge, bonded through a thin epoxy layer to an underlying substrate. The substrate is modeled either as a homogeneous solid or as a re-entrant auxetic lattice with identical external dimensions. This configuration allows us to isolate the geometric

influence while preserving boundary and material constraints.

The simulation domain was discretized with structured meshing, and coupled multiphysics was solved using frequency-domain analysis. Material properties were selected to match fabrication-ready candidates, and mesh refinement was applied in regions of high field gradient to ensure numerical stability.

The distribution of von Mises stress under 0.5 g base excitation is shown in Fig. 3, evaluated at the first resonant mode for each configuration. The conventional base structure exhibits its first natural frequency near 170 Hz, while the auxetic-integrated harvester shifts this frequency downward to approximately 114 Hz. This observed softening is not incidental—it directly reflects the reduced effective bending stiffness, Eq. (6) EI_{eff} , introduced by the auxetic substrate, as predicted by the homogenized power-law relationship, Eq. (8) $E_{eff} = E_0(1 - \phi)^n$.

Beyond frequency alone, the spatial localization of stress is markedly different. While the

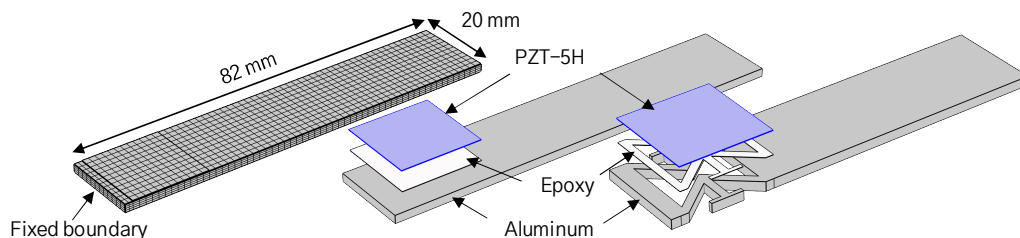


Fig. 2. Numerical setup for the finite element simulation

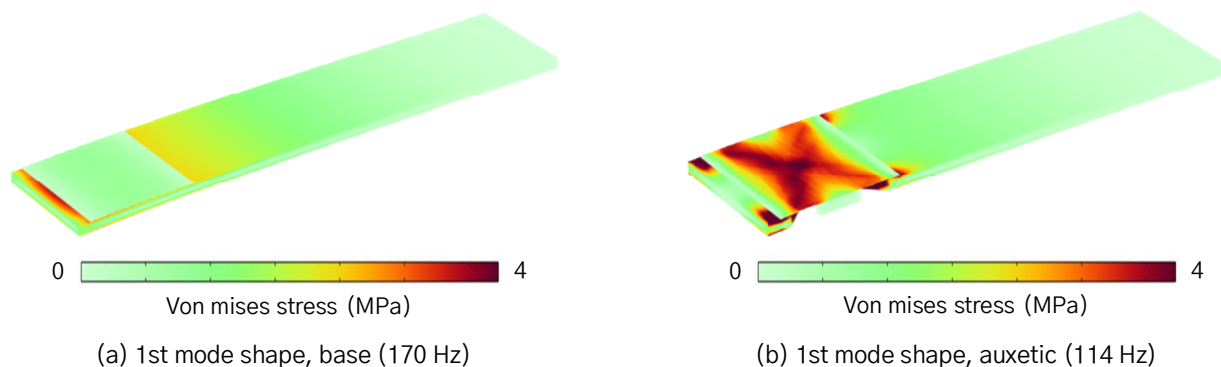


Fig. 3. The distribution of von Mises stress of the base and auxetic structures at the first resonant mode under 0.5 g excitation

base structure shows a broad and modest stress field, the auxetic design concentrates mechanical energy more efficiently beneath the piezoelectric layer. This local intensification of curvature translates—through the governing electro-mechanical coupling—to a larger induced electric field, and thus higher voltage output.

In essence, the auxetic structure acts not only as a mechanical scaffold, but as a geometric amplifier: one that does not merely deform, but sculpts the deformation field itself. The reduction in resonant frequency, when paired with enhanced strain localization, represents a dual gain—more compliant resonance, and more focused energy transfer—both of which are essential for maximizing energy harvesting efficiency.

While the resonant frequency and stress concentration reveal the macroscopic benefits of auxetic substrates, a closer inspection of their directional deformation offers deeper insight. Fig. 4 illustrates the mode shape response of the

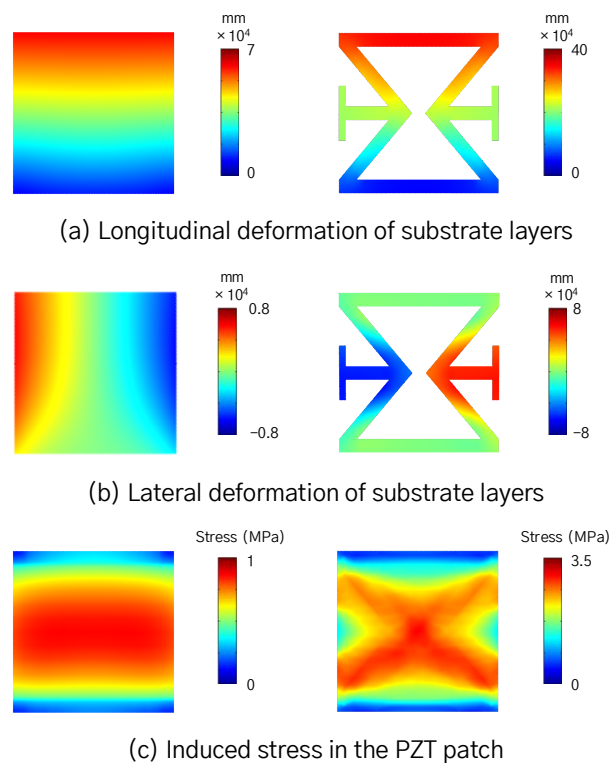


Fig. 4. Longitudinal, lateral deformation of substrate layers, and induced stress in the piezoelectric patch at the first resonant mode

substrate layer in both axial (1-direction) and transverse (2-direction) directions, as well as the resulting stress distribution beneath the piezoelectric patch.

Under first mode excitation, the conventional base substrate exhibits a classic Poisson contraction: as the structure elongates in the axial direction, it simultaneously narrows transversely. In contrast, the auxetic substrate undergoes simultaneous expansion in both directions—a direct manifestation of its negative Poisson's ratio. This geometric behavior is not cosmetic; it realigns the principal strain components, such that the trace strain $\epsilon_{11} + \epsilon_{22}$ becomes maximized rather than diminished.

The outcome is evident in the stress maps. While the base configuration shows modest in-plane stress, the auxetic layer amplifies it nearly fourfold. From the perspective of the piezoelectric constitutive law, where stress drives electric displacement under open-circuit conditions, this amplification is not incidental—it is structurally induced. The substrate no longer passively supports the piezo layer; it actively shapes the strain field to reinforce the electromechanical coupling.

In this sense, auxeticity transcends material classification. It becomes a tool of field engineering—an architectural parameter that tunes not only stiffness and mass, but the very directionality of energy flow within the structure.

The electrical response under varying load resistance is summarized in Fig. 5. While both harvesters exhibit the expected saturation behavior in voltage and a peaked power response, the magnitude of improvement offered by the auxetic configuration is quantitatively significant. The open-circuit voltage increases from 0.74 V to 1.55 V—more than a twofold enhancement—while the peak output power rises from 29.86 μW to 92.07 μW , exceeding a factor of three.

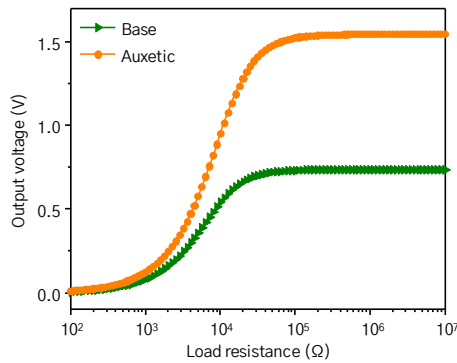
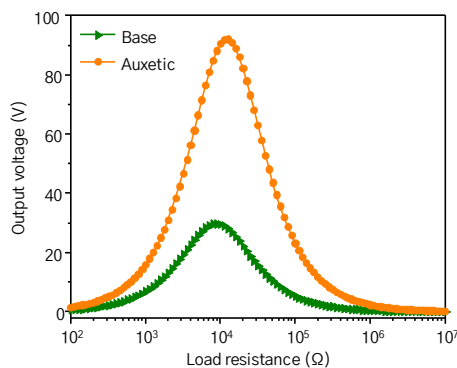
(a) Electrical output voltage (V) vs. load resistance (Ω)(b) Electrical output power (μW) vs. load resistance (Ω)

Fig. 5. Electrical output voltage and power characteristics of base and auxetic harvesters as a function of load resistance

These ratios are not merely statistical; they encode the underlying transformation in how mechanical energy is redistributed and concentrated within the system. The auxetic structure realigns the internal strain field, producing not just greater deformation but more constructive deformation—one that synchronizes with the piezoelectric mechanism to yield amplified electrical response.

That this level of gain is achieved under identical excitation and material constraints highlights the essence of structural intelligence: to achieve more by designing better, not by pushing harder. In a domain where mass, size, and power budget are constrained—as in autonomous naval systems—such a geometry-induced multiplier is not trivial. It is strategic.

The enhancement in output metrics, in light of the previously derived theoretical framework,

closes the loop between structure, physics, and function. Here, the shape of the material dictates the shape of its consequence.

4. Conclusion

This study explored the role of auxetic substrate geometries in enhancing the electromechanical performance of cantilever-type piezoelectric energy harvesters. Through a combination of analytical derivation and finite element simulations, we demonstrated how structural form alone—without altering materials, mass, or excitation—can substantially elevate both voltage and power output. The observed improvements, including a more than twofold increase in open-circuit voltage and a threefold gain in output power, were directly attributable to strain alignment and localized field amplification induced by the auxetic design.

The findings affirm that performance gains in energy harvesting need not rely solely on material innovation or scaling of input energy. Instead, they can emerge from a deeper understanding of how geometry governs internal fields. By reshaping the substrate to reshape the strain, we transform the harvester from a passive receiver into an active participant in its own amplification.

This is, perhaps, the quiet power of structural design: to elicit more from the same, not through force, but through form. In this light, the beauty of materials science lies not merely in composition, but in geometrical tuning—the subtle art of orchestrating deformation to serve function.

Future work may extend this framework to experimental validation and explore the potential of reconfigurable or programmable geometries, where form is not fixed but evolves in response to external demand. Yet even in static structures, this study shows that how a system is shaped can determine what it becomes.

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